On the Multi-Component Pomeron in high energy hadronic interactions

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Abstract

We consider the phenomenon of regge cut splitting, corresponding to the BFKL pomeron into an infinite sequence of regge-poles $I\!P_n$, which happens when one takes into account the running of QCD coupling $\alpha_s(u)$ with scale. The pomeron levels $I\!P_n, \quad n = 1, 2, ...\infty$ have intercepts $j_n(0) \simeq 1 + c/n$ and represent the BFKL-like objects with different mean virtualities $u_i \simeq \ln p_\perp^2/\Lambda^2 \sim n$ on internal lines of corresponding gluon ladders. The first members of the $I\!P_n$ sequence, after adjusting their parameters, may effectively include the nonperturbative part of pomeron, and this way all $I\!P$ components - soft and hard may be treated in a unique way. We also illustrate how the $I\!P_n$ can enter into the phenomenological regge description of some high energy processes.

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Introduction.

Regge phenomenology based on the reggeon diagrams approach [1] allowed to explain and to describe quantitatively various high energy hadronic phenomena and pomeron is one of the main ingredients of such an approach.

In QCD pomeron is usually considered as a specific gluon ladder equipped with some nonperturbative interactions. The BFKL construction [2],[3] refined this in a double logarithmic approximation - here pomeron is a ladder (bound state) of two reggesed gluons. In this approximation QCD coupling α_s is not a running one, and this leads to the transverse conformal-invariance of the parton-gluon motion. As a result, a cut in j-plane at $j = 1 + c\alpha_s$, and not a pole corresponds to pomeron. The mean virtuality of gluons such a pomeron is composed of are not small, and these partons (due to the conformal-invariance) diffuse freely in virtuality $u \simeq \ln p_\perp^2/\Lambda^2$ with growth of rapidity.

But when one takes into account the running of $\alpha_s(u)$ with scale and supplements it with rather general boundary conditions in the infrared region, so that these conditions represent confinement (in fact one stops to move partons to lower u), then the pomeron cut transforms into a sequence of regge poles $I\!P_n$ with intercepts $j_n(0) \simeq 1 + c/n$ [4]-[8].

It is essential that the mean virtualities of gluons, from which the $I\!P_n$ ladders are constructed, grow with n like $< u_n > \equiv < \log k_{n\perp}^2/\Lambda^2 > \sim n$. The first member of this sequence $I\!P_1$ is almost soft, and its properties essentially depend on the details of infrared boundary conditions. Therefore it is possible to expect, that by adjusting these conditions, or directly varying the parameters of the first poles one may effectively take into account the nonperturbative properties of the soft pomeron. There exists a number of reasons that can support such a hope.

It is well known that one can quite good approximate almost all "needed" properties of pomeron (so to explain the most of existing soft high energy data) by simple ladder diagrams with known hadrons $(\pi, \rho, ...)$, where high transverse momenta are cut by vertices (by some form-factors consistent with the low energy hadronic physics). All such models lead to pomeron as a regge pole with some nonlinear trajectory. One would expect the same type answer, while trying to reconstruct the pomeron structure from dominant multipherical processes using unitarity and analyticity in all channels. This suggests that one may take into account all essential nonpertutbative mechanisms using the multiperipheral ladder dynamics with the adequate choice

of pole trajectories and regge vertices.

One may take a slightly different approach and consider the Fock type wave function of fast hadron in the light-cone variables using QCD quark-gluon partons - this is what is contained in BFKL and their generalizations. In this case all nonperturbative effects are connected or with the zero-mode partons in the wave function or with interaction in the final state (after collision) when various QCD strings may be created and decay. These zero-momentum partons interact with other partons (with the arbitrary rapidity) and can essentially modify the structure of the parton wave function at low transverse momenta. In principle one may integrate out these zero-modes, and this induces various additional soft multigluon terms in effective QCD light-cone Hamiltonian. Some of these terms may also correspond to the effective QCD strings stretched between ladder gluons if their transverse separation becomes larger than some critical value. These effects are approximately local in rapidity, due to local color neutralization in the gluon ladder within some interval of rapidity depending on their virtuality. Therefore it seems that the nonperturbative effects shall not change the pole structure of $I\!P$ after such a dressing of the ladder structure.

The other type of reasoning comes from the consideration of high energy scattering in dual models, where the pomeron appears naturally as a cylinder type construction and where the string degrees of freedom already represent the main QCD nonperturbative effects. When such a dual model is considered in AdS_5 space [9],[10] one may at once consider also all main perturbative contributions. If the AdS_5 profile is deformed so to reproduce the α_s running then the attraction of strings to infrared region shall take place and the conformal $I\!P$ splits into $I\!P_n$ - like states. Here the higher $I\!P_n$ states appear as the same sting type state as the soft one, but placed on average at a larger 5-th coordinates, where they become smaller and more virtual.

So one can expect that such a multiple $I\!P$ structure should represent the main perturbative and nonperturbative QCD effects and can be used for universal phenomenological description of various processes in rather large ranges of energy and virtuality [1]. The experimental data support the mul-

[1] On can imagine a slightly different possibility that because of the nonlinear effects in parton evolution and at not too large rapidity the continues regge cut (like in the simple BFKL case) will be not a worse approximation, and here we have the continuous spectrum of pomerons $\mathbf{P}$(u) , labelled by their mean virtuality. Approximately $I\!P_n \simeq \mathbf{P}(u)$ for $n \simeq cu$, but the multireggeon vertices corresponding to these states may differ significantly. Such type approach to soft and hard processes with one continuous pomeron

ticomponent structure of $I\!\!P$ and a number of models with such a $I\!\!P$ where proposed. The simplest model contains pomeron with two components - soft and hard one [12]. Models with more component were also introduced, and they lead (see for example [13]) to good fits for the existing data.

In this paper we discuss some aspects of these problems. We remind the arguments leading to a multi $I\!\!P_n$ pole structure of pomeron and try to elucidate the physics of this phenomenon.

1. The $I\!\!P_n$ states from BFKL equation with running α_s

The infinite sequences of regge poles condensed to the position of born contribution naturally arise in theories where the strength of interaction weakly depends on scale. They correspond to the effective perturbative "ladders" with different mean internal virtuality. Such a phenomenon arises in the asymptotically free $\lambda\varphi^3_6$ theory [14] and in QCD [4] . The intercept of these poles at large n is $j_n(0) \simeq j_{Born}(0) + c/n$, where $j_{Born}(0) = -2$ for $\lambda\varphi^3_6$ and $j_{Born}(0) = 1$ for QCD. The coefficient c defines the density of these poles ; $c^{-1} \sim b$ - the first coefficient of the β function of theory which defines how fast the parton coupling runes with scale. For the first $I\!\!P_n$ states (these components of pomeron, give the main contribution to soft processes) their perturbative intercepts $j_n(0)$ may be essentially shifted due to a nonperturbative effects.

Here we briefly remind how such pomeron states $I\!\!P_n$ appear in the BFKL parton evolution. For the nonrunning α_s there is a transverse scale invariance - as a result we receive the continuous spectrum of regge pole states - the BFKL cut from $j(0) - 1 \ \sim \alpha_s$, representing the upper border of spectrum. For running α_s this cut "splits" into an infinite system of poles accumulating at the point $j(0) = 1$. To see it one may start from the simplest generalization of the BFKL equation to the case of the running coupling α_s :

$$
\begin{aligned}
\frac{1}{\alpha_s(u)}\frac{\partial f(y,u)}{\partial y}) \ &= \ \int du_1 L(u_1 - u) f(y,u_1) \quad , \qquad (1.1) \\
L(u) \ &= \ \int_{-i\infty}^{i\infty} \mathcal{L}(\beta)\, e^{u\beta}\, \frac{d\beta}{2\pi i} \quad , \\
\mathcal{L}(\beta) = \psi(\beta) + \psi(1-\beta) - 2\psi(1) \quad &, \qquad u \equiv \log k_\perp^2/\Lambda^2
\end{aligned}
$$

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was introduced recently in [11].

Then one can approximate the kernel of such an equation by its regular part. This corresponds to the expansion of the kernel $\mathcal{L}(\beta)$ in $\beta = \partial/\partial \log k_\perp^2$ - Laplace conjugated to u around the minimum $\mathcal{L}(\beta) \simeq \mathcal{L}(1/2) + (1/2)(\beta - 0.5)^2\mathcal{L}''(0.5) + \ldots$, and taking only first terms. Then we have from (1.1):

$$\frac{1}{\alpha_s(u)} \cdot \frac{\partial f}{\partial y} \; = \; \delta \; f + B\frac{\partial^2 f}{\partial u^2} \; , \tag{1.2}$$

where for standard QCD with running $\alpha_s(u) \simeq (a + bu)^{-1}$, $a \simeq 0.6$, $b \simeq 0.7$, $\delta = 4N_c \; \ln 2/\pi \simeq 2.6$, $B = 14N_c \; \zeta(3)/\pi \simeq 16$.

The equation (1.2) for the gluon density $f(y, u)$ has the form of a diffusion equation with the parton branching, in which the branching coefficient $\alpha_s(u)\delta$ and the diffusion coefficient $\alpha_s(u)B$ depend on the "coordinate" u. In the same language rapidity plays the role of the time. This analogy is useful for understanding of the behavior of the function $f(y, u)$, and it will be used later.

Note that the nonsingular part of kernel at $\beta \simeq 1/2$ corresponds to a parton chain evolution where the changes of u at individual steps are not large : $\delta u \sim \alpha_s$. The large jumps of u correspond to the region $\beta \sim 1/u$ where $\mathcal{L}(\beta) \sim 1/\beta$, and for which $\delta u \sim \alpha_s u$.

Going to the complex angular momenta in (1.2) as $f_\omega = \int e^{y\omega} f$, $j = 1+\omega$ we come to the equation

$$(a + bu) \cdot \omega f_\omega = \delta f_\omega + B\frac{\partial^2 f_\omega}{\partial u^2} \; , \tag{1.3}$$

which has the Airy form. Its solution can be written as

$$f_\omega(u) = f_\omega(u_0) \cdot \frac{Ai(\; z(u,\omega) \;)}{Ai(\; z(u_0,\omega) \;)} \; , \quad z(u,\omega) = \; \Big(u + \frac{a}{b} - \frac{\delta}{b\omega}\Big)\Big(\omega\frac{b}{B}\Big)^{\frac{1}{3}} \; , \tag{1.4}$$

where $Ai(z)$ is the Airy function, and where boundary conditions at $u = u_0$ are defined by the function $f_\omega(u_0)$ close to the infrared region [2].

In the solution (1.4) the gluon dynamics is, in fact, divided between two factors. One of them is the function $f_\omega(u_0)$ - its singularities in ω represent the "soft" part of the pomeron coming from the region $u < u_0$ and are mainly generated by the nonperturbative mechanisms. The factor $Ai(z(u,\omega))/Ai(z(u_0,\omega))$, oppositely, represents the hard part of pomeron.

[2] Here we reproduce the reasoning given in [7]

The zeros of denominator $Ai(z(u_0,\omega))$ in ω are approximately located at [3] points

$$\omega_n \;=\; \frac{C_1}{n+C_2} \;+\; O(1/n^2), \qquad n=1,2,... \tag{1.5}$$

$$C_1 = \frac{2}{3\pi b}\Big(\frac{\delta}{B}\Big)^{1/2} \;\simeq 0.45 \;\;, \quad C_2 \sim 1$$

The poles with $n \sim 1$ correspond to relatively soft processes ; they become harder and harder when n grows. The order of mean logarithms of a transverse momenta (virtuality) in BFKL ladder, corresponding to n-th pole, are

$$< u_n > \;\;\simeq\;\; c_3\; n \;\;, \qquad c_3 \simeq \frac{3\pi}{4}\Big(\frac{B}{\delta}\Big)^{1/2} \;\simeq 5$$

and they grow fast with n. The distribution of u in $I\!\!P_n$ states around the mean value $< u_n >$ is wide , also of order of $< u_n >$. This may be directly seen from the solution (1.4).

It is essential that the $O(1/n^2)$ and higher corrections to ω_n in (1.5) are u_0-dependent (see (2.5)) - these terms define the slope of $I\!\!P_n$ trajectories. For the first poles (minimal $n = 1$, and partially $= 2$), these corrections, also depending on the boundary condition at u_0, may in considerably change the $\sim C_1/n$ positions of $I\!\!P_n$. This enables to adjust the values $f_\omega(u_0)$ and u_0 in such a way that the $I\!\!P_1$ coincides with the "soft" pomeron, and in this way take into account all main nonperturbative effects.

2. Simple quantum-mechanical model for the $I\!\!P_n$ states

The following analogy is useful to make the structure of the $I\!\!P_n$ states more evident [7]. If we change $y \to iy$ in Eq.(1.2) we become the Schroedinger equation for one-dimensional motion of a "particle" with the coordinate u varying in the interval $u_o < u < \infty$ and y playing the role of time. The Hamiltonian of this motion, as follows from Eq.(1.2), is given by

$$E \;=\; V(u) + \;\hat{p}^2/2m(u) \;\;, \tag{2.1}$$

where the momentum $\hat{p} \;=\; i\hat{\beta} \;=\; -i\partial/\partial u$. The potential and the u coordinate dependent mass in (2.1) are :

$$V(u) = -\delta\alpha_s(u) \simeq -\delta/(a+bu) \;; \qquad m(u) = (2B\alpha_s(u))^{-1} \simeq (a+bu)/2B \tag{2.2}$$

[3] The entire function $Ai(z)$ has zeros only at negative z at points $z \simeq -2.33, \;-4., \;-5.5, ...$ Its asymptotic form at $-z \gg 1$ is $Ai(-z) \simeq \pi^{-1/2} z^{-1/4} \sin(\frac{2}{3} z^{3/2} + \pi/4)$.

In a case of a constant $\alpha_s(u)$ we have a free motion in u. So, when we put the corresponding "particle"(it is the Pomeron ladder) at initial time $y=0$ to some position u_1 (it is with initial $I\!\!P$ transverse size $\sim \exp(-u_1)$) then at the later time there will be simply a spreading of a wave packet, described by the Green function of free motion - it corresponds to the "standard" BFKL behavior (Gaussian spreading in virtuality u).

But for the running $\alpha_s(u)$ the motion is not free - we have long range (in u) attractive forces $\sim 1/u^2$, acting to the direction of small u.

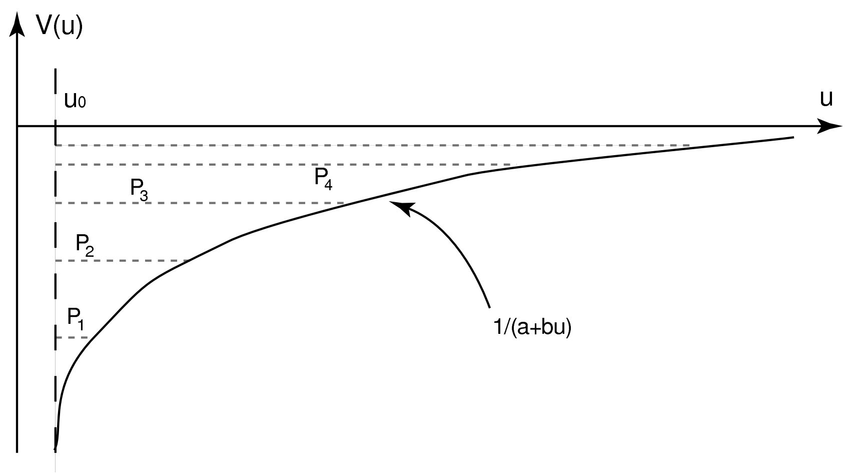

Figure 1: Qualitative form of potential $V(u)$ and of the $I\!\!P_n$ levels.

In this potential, due to one-dimensionality of the motion, the infinite "Coulomb-like" series of bound states exist. For the high lying levels their motion is on average located at large "distances" $u \gg u_0$ where the motion is quasiclassical. Solving the expression (2.1) for E relative to p we can write the standard quasiclassical quantization conditions

$$n\,\pi \;=\; I_n\;, \quad I_n \;=\; \int_{n_0}^{u_{max}(\varepsilon)} \; p(u)\;du\;, \;\; p^2 = 2m(u)(\varepsilon - V(u))\;, \qquad (2.3)$$

where $u_{max}(\varepsilon) \;=\; (\delta - a\varepsilon)/b\varepsilon$. Than, using (2.2) we find from here

$$I_n \;=\; \pi\,\frac{\omega_0}{\varepsilon}\,\Big(1-\varepsilon\,\frac{a+b\;u_0}{\delta}\Big)^{3/2}\;, \quad \omega_0 \;=\; \frac{2}{3\pi}\Big(\frac{\delta}{b}\Big)\Big(\frac{\delta}{B}\Big)^{1/2}\;,$$

and the equation for the position of ε-levels takes the form

$$\varepsilon \;=\; \;=\; \frac{\omega_0}{n}\,\Big(1-\varepsilon\,\frac{1}{\delta\alpha_s(u_0)}\Big)^{3/2} \qquad (2.4)$$

From here for large n we find the simple solution for positions of $I\!\!P_n$ levels

$$\varepsilon \simeq \omega_0 \frac{\alpha_s(u_0)}{n\alpha_s(u_0) + C_2} = \frac{\omega_0}{n + C_2(a + bu_0)} , \quad C_2 = \frac{3\omega_0}{2\delta} , \tag{2.5}$$

which has the same structure as (1.5). But even for minimal $n = 1$ this estimate may be acceptable, because parameters entering (2.5) for realistic QCD are approximately

$$\omega_0 \simeq 0.3 \quad , \quad C_2 =\simeq 0.172 .$$

The value of $\alpha_s(u_0) \simeq 0.3 \div 0.6$, entering (2.5) should be taken at such $u_0 \sim 2 \div 3$, when we go to strong nonperturbative dynamics and $\alpha_s(u_0)$ become frozen. So, if we choose the value of the nonperturbative border at $\Lambda_0 \simeq 0.5 \div 1\ GeV$, and $\Lambda_s \simeq 0.15 GeV$, which corresponds to $u_0 \simeq 3 \div 5$, we become from (2.5) for $I\!\!P_n$ intercepts the estimate $\omega_1 \simeq 0.2$, $\omega_2 \simeq 0.12$, $\omega_3 \simeq 0.085$, ...
The wave functions $\psi_n(u)$ of P_n states

$$\psi_n(u) \sim \frac{1}{\sqrt{n}} \exp\Big(i \int p(u)\ du\Big) \tag{2.6}$$

are spread at large n for large interval of virtualities $\sim u \sim n$, inside the potential well. But $\psi_n(u)$ are rather smooth and even at large n are influenced by the "repulsive" boundary conditions at u_0.

Therefore one may try to adjust the potential $V(u)$ near u_0 in such a way to "move" the nonperturbative contributions in the pomeron state from ϕ_ω directly into $I\!\!P_1$. After that all singularities of ϕ_ω in (1.4) are located at $\omega \leq 0$.

Because confinement (as we are understanding it now) must strongly restrict the color particles motion at large distances we simply suppose that effective potential $V(u)$ becomes very big and positive at $u < u_0$. It means that the levels $I\!\!P_n$ will always remain discreet, and only the position of few first terms $I\!\!P_n$ must by adjusted to take into account nonperturbative effects.

From (2.4) it is simple to find the slope of $I\!\!P_n$ trajectories defining them from the reaction of levels on the shift of u_0 boundary conditions

$$\alpha'_n(0) \simeq -\frac{1}{\Lambda_0^2} \frac{\partial \varepsilon_n}{\partial u_0} \simeq \frac{1}{\Lambda_0^2} \frac{C}{(n)^2} \quad , \quad C \simeq 0.25 \quad , \quad u_0 \equiv \log \Lambda_0^2/\Lambda_s^2$$

With the same method we can find the full regge trajectories for $I\!P_n$, associating the shift of boundary from $u_0 = \log(\Lambda_0^2/\Lambda_c^2$ to the new boundary $\log((\Lambda_0^2 + |t|)/\Lambda_c^2$ with growth of $|t|$ from the zero value. The result is :

$$j_n(t) \ \simeq \ 1 \ + \ \frac{\omega_0}{n + C_2 \ (\alpha_s(\hat{u}))^{-1}} \quad , \qquad \hat{u} = \log \frac{\Lambda_0^2 + |t|}{\Lambda_c^2} \tag{2.7}$$

Such an expression appears because the position of Pomeron pole at some $-t$ is approximately the same as its position, when the region of the transverse motion of intermediate gluons is restricted by the condition $u > \log{(-t)}$. Note, that for large u_0 the region of motion $u_{max}(n) - u_0$ is concentrated at large u also for $n = 1$, and therefore the corrections to quasiclassical expressions for $\alpha_n(t)$ may be small for large $-t$ even for the ground level $I\!P_1$.

3. The higher order corrections to $I\!P_n$ states

The BFKL equation is formally valid in rather specific conditions, corresponding to the main logarithmic approximation in $\alpha_s \log s$. One may expect that the higher in α_s corrections and the nonperturbative contributions can induce additional properties of pomeron not seen in lowest approximation when only running of α_s is taken into account. They may be :

* Corrections to $I\!P_n$ parameters and the mixing of $I\!P_n$ with multigluon states.
* Nonperturbative effects.
* Nonlinear terms in the gluon cascade evolution.

Let us discuss them briefly. The higher α_s perturbative terms can in particular essentially modify the regular part of the BFKL-kernel [3]. But it probably will not lead to a qualitatively new effects and can be taken into account by adjusting parameters of $I\!P_n$ trajectories and values $I\!P_n$ of vertices. Already in the conformal approximation there are the multigluon ladders [15] which give additional regge singularities with a vacuum quantum number. Probably every such an object will also split into the complicated ensemble of discreet levels (it will have the fine structure like the $I\!P_n$), after going from the conformal approximation by insertion of varying $\alpha_s(u)$. The multigluon state can mix with $I\!P_n$ state when one abandons the conformal approximation, because the $I\!P_n$ state "consists" of the reggesed gluons, which by themselves are the multigluon states. Such a mixing may appear in higher in $\alpha_s(u)$ terms and the corresponding vertices are of the same type as $3I\!P$ and higher

$I\!\!P$ vertices. Corresponding contributions are of approximately the same form as the self-energy corrections to $I\!\!P$ propagators and vertices, and probably can be taken into account by small "phenomenological" shifts of Δ_n.

The nonperturbative contributions to $I\!\!P$ was partially discussed in Introduction and it needs a separate detailed investigation. Here we make only one note. At small t there can be also essential the mixing of a $I\!\!P_n$ with states composed from light quarks in specific configurations corresponding to 2π states. This is connected to the existence of the $\langle \bar{q}q \rangle$ condensate and manifests in the large contribution of 2π exchanges in multiperipheral diagrams connected to pomeron. By this mechanism the intercepts of the resulting states j_n may be shifted on $\delta j_n(0) \sim \pm\Lambda^2 < x_\perp^2 >_n \sim \pm \exp{(-cn)}$. This shift may be essential only for the first intercept $j_1(0)$, representing the most soft part of pomeron. As a result it can even be (and the data gives some support for this) that $j_1(0) < j_2(0)$.

The other extension of BFKL is connected with fusion of gluons, which becomes more and more essential at large y, and which leads to the gluon saturation at different virtuality scales $u \sim n$. The simplest model of gluon cascade describing all these phenomena result from the differential BFKL equation (1.2) supplemented by the nonlinear terms, corresponding to $3I\!\!P$ (and all higher) diagrams

$$\frac{1}{\alpha_s(u)} \cdot \frac{\partial f(u,y)}{\partial y} \;=\; \delta\ f + B\frac{\partial^2 f}{\partial u^2} - r(u) f^2(u,y) + ..., \qquad (3.1)$$

where $r(u)$ is proportional to the $3I\!\!P$ vertex at virtuality u. The nonlinear corrections in (3.1) correspond to splitting and gluing of pomerons, and the perturbative in $r(u)$ solution for f may be represented by the reggeon diagrams, containing pomerons coming from the linear equation. A more accurate than (3.1) equations [16] contain in fact the same physical information, and therefore we do not need to enter here into details.

It seems therefore plausible that most higher α_s corrections, which in particular make amplitudes unitary in all channels, may be taken into account by summing of contributions of various reggeon diagrams with all $I\!\!P_n$ states. The properties of these diagrams can be summarized (as a simple generalization of one pomeron case) by the introduction of the effective Lagrangian

for the $I\!P_n$ reggeon field theory

$$\mathcal{L} \ = \ \sum_n \ \Big(\ \Psi^+_n \frac{\overleftrightarrow{\partial}}{2\ \partial y} \Psi_n \ + \ \Delta_n \Psi^+_n \Psi_n \ \ + \ \alpha'_n \ \vec{\partial_\perp} \Psi^+_n \ \vec{\partial_\perp} \Psi_n \ + \qquad (3.2)$$
$$+ \ \left(\Psi^+_n J_n + J^+_n \Psi_n\right) \Big) \ + \ \ i \sum_{mnk} \ r_{mnk} \ \left(\Psi^+_m \Psi^+_n \Psi_k + \Psi^+_k \Psi_m \Psi_n\right) \ + \ ...$$

containing various $I\!P_n$ interactions, where $\Psi_n(y, x_\perp)$ is the $I\!P_n$ -the reggeon field, r_{mnk} - $3\,I\!P$,... vertices between the corresponding $I\!P_n$ states, J_n - are the external currents representing colliding particles(nuclei), the simplest one are $J_n \simeq g^{(a)}_n \delta(y - Y)$, $J^+_n \ = \ g^{(b)}_n \delta(y)$. From (3.2) on can in usual way construct all reggeon diagrams with $I\!P_n$.

The parton saturation in terms of reggeon diagrams corresponds to transition to a state in which reggeon field operators $\Psi_n(b, y)$ have the nonzero vacuum expectation value $\langle\Psi_n\rangle \sim \Delta_n/r_{nnn}$. The $\langle\Psi_n\rangle$ condensation (saturation) takes place in the bubble that grows with y around external sources J_n. As a result the new $I\!P_n$ fields $\psi_n = \Psi_n - \langle\Psi_n\rangle$ enter into the effective Lagrangian in such a $\langle\Psi_m\rangle$-bubble with different parameters $\Delta_n, \alpha'_n, r_{n\ m\ m}$, etc, depending on the properties of this medium.

Such a behavior is typical for the supercritical reggeon theory and leads to the Froissart asymptotics of cross-section. Here the Froissart disk is a transverse region filled with condensed $\langle\Psi(b, y)\rangle$. This picture can be generalized to the multi - $I\!P_n$ reggeon field theory (see Section 7).

To understand the range of applicability of the regge approach in the case of supercritical pomeron one can estimate the average rapidity intervals $\langle y_n\rangle$ on the $I\!P_n$ lines in general complicated reggeon diagrams, essential at asymptotic energies, and find how $\langle y_n\rangle$ changes with growth of the full rapidity Y. Its value is evidently connected with amplitudes of pomerons splitting and joining defined by their intercepts and values of r_n vertices.

If we imagine such a reggeon as a particle propagating in a medium composed from other similar pomerons, then a simple estimate of time (rapidity) interval between interactions of this $I\!P_n$ particle is

$$\tilde{y}_n \sim \frac{1}{r^2_n \rho_n} \ \ , \qquad (3.3)$$

where $\rho_n \sim$ to a density of pomerons around the considered pomeron, and entering relation r^2_n is $\sim$ to the probability the considered pomeron to interact

with neighbor pomerons in the unit of time y. In (3.3) we also supposed that the main $I\!P_n$ interactions are with pomerons of the same size, and the $3I\!P$ vertex $r_n \simeq r_{nnn}$.

If the full rapidity interval Y is so that we yet are far from the $I\!P_n$ saturation scale, the value of ρ_n is small and it gradually grows with Y. When the full energy riches the saturation scale for $I\!P_n$, then the mean $I\!P_n$ density $\rho_n \sim (\Delta_n/r_n)^2$, and it stops to grow with Y, so that for the average pomeron "lifetimes" in these conditions we have $\tilde{y}_n \sim 1/\Delta_n^2$.

Because the average y-distance between the nearest steps on the effective $I\!P_n$ pomeron ladder is $\sim 1/\Delta_n$, the mean $I\!P_n$ pomeron line even in the saturated medium will have inside it $\sim 1/\Delta_n \sim n/\alpha_s \gg 1$ "ladder" steps. And therefore the $I\!P_n$ quasi-particles can be consistently used also inside the Froissart disk.

Now let us consider the multi-reggeon vertices with attached $I\!P_n$ states. At first sight the value of various multi-$I\!P$ vertices for $I\!P_n$ may by estimated using vertices describing the joining of many pomerons of the BFKL type at a given virtualities.

So, for example, if we consider the $3\,I\!P_n$ vertex with all three $n_1 \simeq n_2 \simeq n_3$ of the same order, then we have - $\langle u_1 \rangle \simeq \langle u_2 \rangle \simeq \langle u_3 \rangle \sim n_i$. Then on dimensional grounds it may be expected that $r_{3p}(u,u,u) \sim \Lambda_c^{-1} \exp(-u)$. This would be correct if r_{3p} represents the vertex in which one joins pomeron states with definite u_i , that is with definite transverse sizes. But the $I\!P_n$ states contain the superposition of transverse distances with amplitudes ψ_n , and therefore $r_{3p}(n_1 n_2 n_3)$ is proportional to overlapping integrals

$$\int du_1 du_2 du_3 \;\psi_{n1}(u_1)\psi_{n2}(u_2)\psi_{n3}(u_3)\;\tilde{r}(u_1,u_2,u_3)\;,$$

where $\tilde{r}(u_1 u_2 u_3) \sim \exp(-cu_i)$ corresponds to the internal part of the r_{3p} vertex. Because $\psi_n(u)$ are not small at low u, it follows from (2.6) that $\psi_n(u \sim 1) \sim 1/\sqrt{n}$, so that the value of r_{3p} vertices can be estimated as

$$r_{3p}(n_1 n_2 n_3) \;\sim\; \Lambda_c^{-1}\;(n_1 n_2 n_3)^{-1/2}$$

All other mulipomeron vertices with definite n_i have probably the same structure - i.e are not too small.

But near the saturation region the value of vertices may drastically change. So if the first m states $I\!P_n$ are already saturated the low u part of all $\psi_n(u)$

are cut up to $u < \exp(cm)$. And it will lead to the estimation

$$r_{3p}(n_1 n_2 n_3; m) \ \sim \Lambda_c^{-1} \exp(-cm) \ .$$

The regge description, especially with many $I\!\!P_n$, contains a large number of "phenomenological" parameters such as $I\!\!P_n$ intercepts Δ_n and slopes α'_n, entering $I\!\!P_n$ trajectories $j_n(t)$, the $I\!\!P_n$ interaction vertices r_{mnk}, and the inclusive vertices γ_n. In principle all these quantities "may" be calculated directly from QCD as a function of α_s and some nonperturbative QCD parameters. But now it is probably too early to hope that it can be done in a quantitative way.

5. $I\!\!P_n$ states and the parton picture

The parton picture corresponding to BFKL pomeron is relatively simple. The mean parton configurations for a fast colorless hadron are produced by the branched gluon cascade in y with average number of steps $\sim \alpha_s y$. This leads to an exponential growth with y of the number of low energy partons(gluons).

In the case of not running α_s the parton "motion" in its virtuality $u = \log k_\perp^2/\Lambda^2$ is conformal-invariant, and at growth of y the free random motion of partons virtuality $u_i(y)$ takes place. As a result the lowermost partons have the virtualities $u_i \sim \sqrt{\alpha_s y}$ on average.

In the case of running $\alpha_s(u)$ the diffusion in u takes place with the additional drift in the direction of small u. The corresponding distribution in virtuality of bottom-partons may be represented as the approximate solution of Eq.(1.2) in such form:

$$f(u,y) \ \sim \ \frac{f_0}{y} \ \exp\Big(\ \delta \ y \ \alpha_s(u) - \frac{(u-u_0)^2}{4 \ B \ y \ \alpha_s(u)} \ \Big) \ , \quad \alpha_s(u) = \frac{1}{a+bu} \qquad (5.1)$$

From (5.1) it evidently follows that at large y the mean virtuality is

$$< u > \ = \ \int u f(u,y) du \Big/ \int f(u,y) du \sim const(y) \sim 1 \ ,$$

and does not grow with y. But the mean $\langle k_\perp^2 \rangle$ will grow as $\exp\left(2\sqrt{yB/b}\right)$, and this reflects the u-motion of partons in the $I\!\!P_n$ states .

A $I\!\!P_n$ state does not directly correspond to a definite parton configuration of fast hadron, but only to a superposition of specific choices of parton

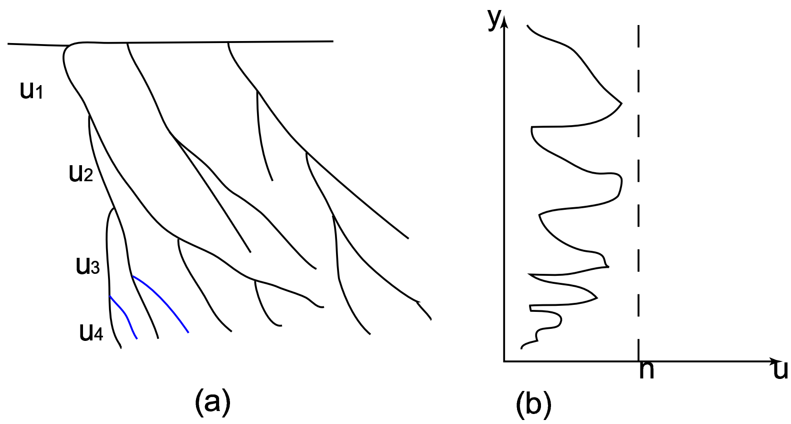

Figure 2: (a) Parton state - Gluon cascade, with dedicated parton branch. (b) The u-motion of partons on the selected parton branch.

branches which interact with a target [4]. The $I\!\!P_n$ approximately corresponds to parton branches interacting with the target with the mean bottom virtuality $u \sim n$. For a large y the value of virtuality $u_i(y)$ along such a parton branch "oscillates" between $u \sim 1$ and $u \sim n$. The period in y of such u-oscillations is $\tau_n \simeq 3n^2$. These values directly correspond to the splinting between adjacent $I\!\!P_n$ levels $\tau_n^{-1} \simeq |\Delta_n - \Delta_{n+1}|$.

From here it follows some evident limitation for using $I\!\!P_n$ states in phenomenology, because τ_n grow rather fast, and the currently experimentally reachable rapidities are limited $y < 20 \div 25$. So only the first few $I\!\!P_n$ states can be distinguished, and all higher ones act as a single hard regge singularity at $j \simeq 1$.

The other limitation is connected with the fusion of gluons, which becomes more and more essential at large y, and the transition to the gluon saturation at the different virtuality scales $u \sim n$. The simplest model of gluon cascade describing all these phenomena results from the differential BFKL equation supplemented just as in (3.1) by the nonlinear terms.

Neglecting in (3.1) the diffusion term $\partial^2 f/\partial u^2$, we become the simple

[4]Here the situation is similar to the case for other regge poles, for example, non-vacuum, in that case the interaction with the target "selects" the branch of a parton cascade through which the corresponding quantum number is transported from the valence quark region.

solution

$$f(u,y) \;=\; \frac{\exp\left(\delta y \alpha_s(u)\right)}{f_0(u)^{-1} \;+\; \delta^{-1}\, r(u)\, \Big(\exp\left(\delta y \alpha_s(u)\right) - 1\Big)} \quad\Longrightarrow\quad \frac{\delta}{r(u)}\;, \tag{5.2}$$

$$f_0(u) \;=\; f(u, y=0)$$

at $y \Longrightarrow \infty$, so that for every virtuality the gluon density stops to grow after rapidity reaches some critical value depending on u. This parton saturation for the virtuality $u \sim n$ takes place at

$$\tilde{y} \;\geq\; \frac{1}{\delta\alpha_s(u)} \log\frac{\delta}{f_0(u) r(u)} \quad\sim\quad \frac{b}{\delta}\, u^2 + u c(u) \quad,\quad u \simeq n$$

The last expression is the estimation for large u, where we supposed that $\log r(u)^{-1} \sim u$. So with growth of y the consecutive saturation of parton densities at higher and higher scales $\tilde{u}(y) \sim \sqrt{y\delta/b}$ takes place, and it may be interpreted as a consecutive saturation of densities of the $I\!\!P_n$ pomerons with $n \sim \tilde{u}(y)$.

6. Regge phenomenology with $I\!\!P_n$

The contribution of higher $I\!\!P_n$ states may be essential for a description of various reactions in the regge approach, especially in the cases when some of the transverse momentum transfers become large. But, as it was mentioned above, the separation of contributions of $I\!\!P_n$ at "real" and not at asymptotic $y \to \infty$ energies cannot be done. So, possibly, at all experimentally accessible energies the reasonable approximation may contain a short sequence of distinguishable $I\!\!P_n$ states $n = 1, 2, ...m$, $m \sim 2 \div 3$ and the rest $I\!\!P_n$ $n > m$ will look as one indistinguishable hard pomeron singularity $\tilde{I\!\!P}$ at $j = 1$.

It seems that one can base a reasonable phenomenology for all main high energy processes using such a truncated system of $I\!\!P_n$ states. It can look much simpler than approaches based on a direct use of QCD variables. From one side, it hides the really essential degrees of freedom, but from the other side, the unitarity conditions in all channels are explicitly under control.

In the next Section we consider briefly the asymptotic Froissart behavior to illustrate the process of parton saturation at various virtuality scales, and how it arises from $I\!\!P_n$ states.

7. Saturation and Froissart behavior

The Froissart type asymptotics always is expected [17] for supercritical reggeon theories - in QCD we have directly this case. The usual way to such a behavior comes from an eikonal summation of pomeron exchanges, and it leads to the inelastic cross-sections $\sigma(y) =\simeq \pi R^2(y)$. In this case the radius of the soft Froissart disk in the impact parameter space $b_\perp$ grows with y like $R(y) = vy$, where the transverse "velocity" of growth $v = 2\sqrt{\alpha'\Delta}$. This mechanism is rather primitive - pomerons do not interact one with another - and as a result the effective parton (and pomeron) density exponentially grows with y inside the "disk", and we have not a saturation but only a screening of partons. But occasionally this simple model gives many correct predictions. It leads to a black and not a gray Froissart disk, and the corresponding parton model is boost-invariant. Probably, the main defect of this model is that it is soft, but this disadvantage may be cured if we generalize the model to the multi $I\!\!P_n$ case and take into account the pomeron interactions which lead to the parton saturation.

This looks quite easy. Taking for the $I\!\!P_n$ contributions in the usual factorized form

$$\chi_n(b_\perp, y) \;=\; i\,\frac{g_n \cdot g_n}{\alpha'_n y_1}\;\exp\Big(\,\Delta_n y_1 - b_\perp^2/4\alpha'_n y_1\,\Big)\;, \quad y_1 = y - i\pi\Delta_n/2\;, \quad (7.1)$$

we have for the full $S(b_\perp, y)$-matrix and amplitude the eikonal like [5] expressions

$$\begin{aligned} S(b_\perp, y) \;&=\; S[\{\chi_n\}] \;=\; \exp\Big(\,i\sum_n \chi_n(b_\perp, y)\Big) \;\simeq\; \theta(b^2 - R_1^2(y))\;, \quad (7.2)\\ A(b_\perp, y) \;&=\; i(1 - S(b_\perp, y)) \;\simeq\; i\theta(R_1^2(y) - b_\perp^2)\;, \end{aligned}$$

where $R_1(y) = y\sqrt{\alpha'_1\Delta_1}$ is the soft disk $\mathcal{F}_1$ radius. This soft disk contains also a chain of more hard disks $\mathcal{F}_n$ with smaller radii $R_n^2(y) \;=\; \alpha'_n\Delta_n y^2 \equiv v_n^2\, y^2$ and large average parton(pomeron) virtuality $u_n \sim n$. The hard disks $\mathcal{F}_n$ are only slowly reflected in total cross sections - they simply make the internal

[5]Note that there are no reasonable arguments that the full contribution from the exchange of noninteracting $I\!\!P$ should take the eikonal form $S = \exp i\chi$. The correct form of $S[\chi_n]$ may be completely different because the contribution from diffractive jets may be very big; for example S can behave like $(1 - i\chi)^{-1}$. But this will not change the general structure of the Froissaron - the full picture of embedded disks remains the same.

part of the Froissart disk more dark. But if we consider the inclusive cross-section for particles with a high mass or other events with high virtuality the created particles come mostly from the corresponding hard disks $\mathcal{F}_n$. This in particular leads to the growth of mean transverse momenta of secondary particles with energy.

Let us examine how this simple picture changes when we take into account the $I\!\!P_n$ pomeron interactions. Consider the soft disk $\mathcal{F}_1$ composed from $I\!\!P_1$. Near the border of $\mathcal{F}_1$ at $b \simeq v_1 y$ the $I\!\!P_1$ density is ~ 1, and here the reggeon interaction is not especially essential. Just the rate of parton splitting in this region defines velocity v_1 of $\mathcal{F}_1$-disk growth with y. Therefore in the first approximation the effect of pomeron fusion are essential only in the inner parts of $\mathcal{F}_1$-disk where we may also neglect the transverse (in $b_\perp$) motion of $I\!\!P_1$-reggeons and use for its density the expression (5.2). This gives the value of the saturated $I\!\!P_1$ density inside $\mathcal{F}_1$ of order $f_1 \simeq \Delta_1/r_1$. It must be corrected only near the border of $\mathcal{F}_1$ where density changes from f_1 to values of the order ~ 1. The width of this strip is $\sim \sqrt{\alpha_1'/\Delta_1}\log(\Delta_1/r_1)$.

Now consider the higher $\mathcal{F}_n$-disk and ask by what a way the density of $I\!\!P_n$ at large distance $b_\perp$ is generated. Various mechanisms may operate. The one is connected with the direct growth of $\mathcal{F}_n$ disk from $b_\perp = 0$, but it is very slow and leads to the small radius of saturated disk $R_n \sim y/n^3$. The most effective mechanism [18] that transports the high virtuality partons to larger $b_\perp$ is connected with the local growth of $\mathcal{F}_n$ density from the soft saturated $\mathcal{F}_1$ disk whose radius has already reached this value of $b_\perp$. To describe it we may use the expression (5.2) for $f(u, y)$ where we change $y \to y - b_\perp/v_1$ and use for $f(u, 0) \sim \exp(-u)$. From this we conclude that the saturation at scale u and distance $b_\perp$ is reached at $y = b_\perp/v_1 + (2b/\delta)u^2$. This corresponds to the saturated $\mathcal{F}_n$ disk radius

$$R_n(y) \ \sim \ v_1 y - \langle u^2 \rangle_n \ (2bv_1/\delta) \ = \ R_1(y) - n^2\lambda \ , \qquad \lambda = 2v_1 b c_3/\delta$$

It follows from here that the maximal saturated virtuality in the center of $\mathcal{F}$ disk is $u \sim \sqrt{y}$. One can also define approximately the width δR_n of the $\mathcal{F}_n$ border from the condition that the reggeon density changes from the saturated value $f_n \sim \Delta_n/r_n$ up to ~ 1. From this we become $\delta R_n \sim n\, v_1 \Delta^{-1}$.

Note that the value of the transparency of full $\mathcal{F}$ disk should not depend essentially from the pomeron interactions. This is because the main contribution to $\mathcal{F}$-transparency, which is $\sim |S(b_\perp, y)|^2$, comes from the components of the fast hadron wave function without a parton cloud and

correspondingly without $\mathcal{F}$ disk. An estimation of this probability gives the boost-invariant answer $|S(b_\perp, y)|^2 \sim \exp(-c(b_\perp)\ y)$. This quantity enters various cross-sections, and it would be interesting to check its properties experimentally at the most possible energies, because this may reflect essential elements of parton dynamics.

Conclusion

The aim of this article was to discuss the main physical properties of $I\!P_n$ states and to understand if the use of $I\!P_n$ in regge phenomenology may be natural and useful. Our answer is more positive than negative It seems that the approach based on the multi-$I\!P_n$ sequence may adequately represent most essential aspects of high energy interactions - from QCD perturbative to nonperturbative ones, in particular a growth of a mean virtuality with energy and parton density saturation. Also the purely asymptotical picture of the Froissart limit as represented by a system of nested disks filled by the $I\!P_n$ condensate looks quite reasonable.

It is essential that by using the regge approach with multiple $I\!P_n$, instead of working directly QCD degrees of freedom, we may expect that the unitarity restrictions on amplitudes from different crossing channels are almost automatically taken into account. This is a big advantage from the phenomenological point of view. But even at the fundamental theoretical level it may be more simple to calculate once the values of regge vertices for $I\!P_n$ from QCD, than do all this from the beginning for every amplitude.

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